A.A. 2010/11 – 3° COMPITINO – 18/4/2011

Titolo nota 17/04/2011

1) Supponiamo che l'acqua riesca a fornire tutto il calore necessario al ghiaccio:

$$m_{gh} c_{gh} (T_{fus} - T_{gh}^i) + m_{gh} \lambda_{fus} + m_{gh} c_{H_2O} (T_e - T_{fus}) + m_{H_2O} c_{H_2O} (T_e - T_{H_2O}^i) = 0$$

$$\Rightarrow T_e = \frac{m_{H_2O} c_{H_2O} T_{H_2O}^i + m_{gh} c_{H_2O} T_{fus} - m_{gh} c_{gh} (T_{fus} - T_{gh}^i) - m_{gh} \lambda_{fus}}{(m_{gh} + m_{H_2O}) c_{H_2O}} \quad (*)$$

Sostituendo i valori dati (ed usando $T_{fus} = 273.15\,K$ e $c_{H_2O} = 1 \frac{cal}{g\,K} = 4.186 \frac{J}{g\,K}$) si ottiene, per il punto a) ($m_{gh} = 1.5\,kg$)

$$T_e = 241.7\,K < T_{fus}$$

$\Rightarrow$ Il ghiaccio non può essere fuso interamente.

Si assuma che la massa di ghiaccio fusa sia pari a m_x (e che quindi la temperatura finale sia T_{fus})

$$\rightarrow m_{gh} c_{gh} (T_{fus} - T_{gh}^i) + m_x \lambda_{fus} + m_{H_2O} c_{H_2O} (T_{fus} - T_{H_2O}^i) = 0$$

$$\Rightarrow m_x = \frac{m_{H_2O} c_{H_2O} (T_{H_2O}^i - T_{fus}) - m_{gh} c_{gh} (T_{fus} - T_{gh}^i)}{\lambda_{fus}} =$$

$$= 125\,g$$ ($>0 \Rightarrow$ le assunzioni erano corrette)

$\Rightarrow$ Nel caso a):

$T_f = 273.15\,K = 0\,°C$

$m_{acqua\,totale} = 2.125\,kg \qquad m_{gh\,totale} = 1.375\,kg$

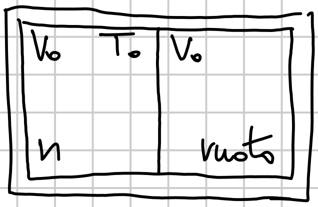
Nel caso b), usando la (*), si trova

$T_e = 277.5\,K = 4.35\,°C \Rightarrow$ tutto il ghiaccio fonde

$\Rightarrow$ Nel caso b)

$T_f = 277.5\,K = 4.35\,°C$

$m_{acqua\,totale} = 2.3\,kg$, $m_{gh\,totale} = 0\,g$

2) 

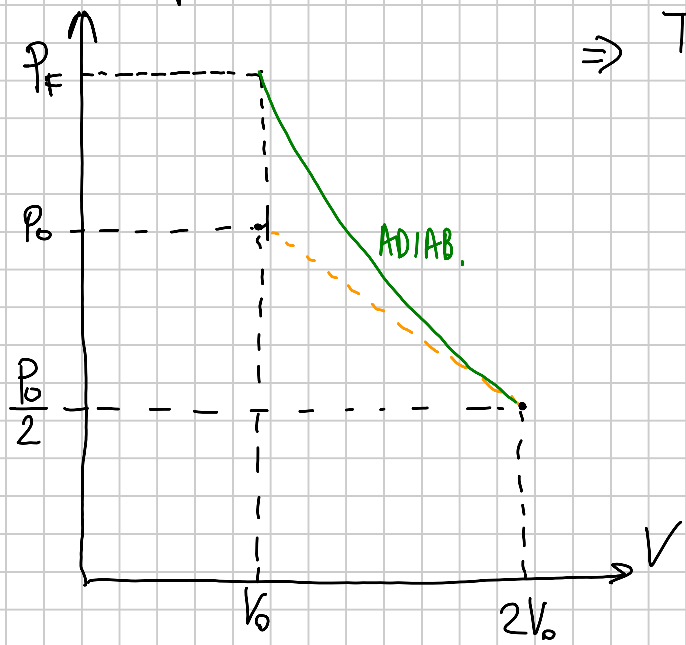
n gas monoatomico

Espansione libera + pistone riportato lentamente nella posizione iniziale

$\Delta S,\ \Delta U = ?$

| Espansione libera → $Q=0,\ W=0 \Rightarrow \Delta U = 0$

$\Rightarrow T_{fin} = T_0,\ V_{fin} = 2V_0,\ P_{fin} = \frac{P_0}{2}$



$\Delta U_{-} = 0$
$\Delta S_{-} = nR\ln 2$

$\frac{P_0}{2}(2V_0)^\gamma = P_F V_0^\gamma$

$P_F = 2^{\gamma-1} P_0$

$\frac{P_0V_0}{nRT_0} = \frac{P_F V_0}{nRT_F} \Rightarrow$

$T_F = \frac{P_F}{P_0} T_0 = 2^{\gamma-1} T_0$

$\Rightarrow \Delta U_{-} = nc_V(2^{\gamma-1}-1)T_0$
$\Delta S_{-} = 0$

$\Rightarrow \boxed{\Delta S_{TOT} = nR\ln 2}$

$$\Delta U_{TOT} = nC_V(2^{\gamma-1}-1)T_0 = \frac{3}{2}nR(\sqrt[3]{4}-1)T_0$$

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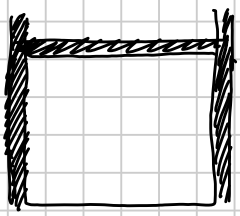
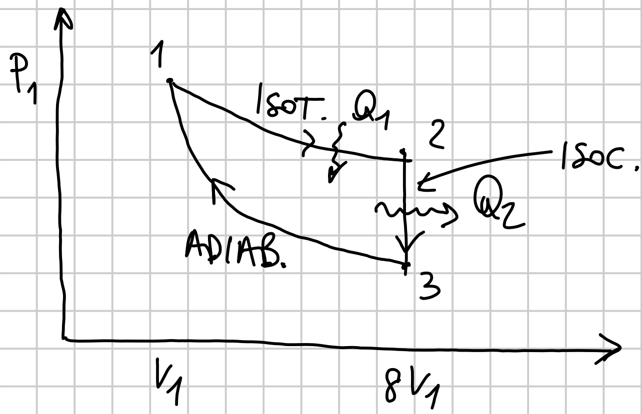
3) 

$P_1,\ V_1$

n=1 monoatomico

$\eta = ?$



a) $\eta = \frac{W}{Q_1} = 1 - \frac{Q_2}{Q_1}$

$P_2V_2 = P_1V_1 \Rightarrow P_2 = \frac{P_1}{8}$

$P_1V_1^\gamma = P_3V_3^\gamma = P_3V_2^\gamma \Rightarrow P_3 = \frac{P_1}{8^\gamma}$

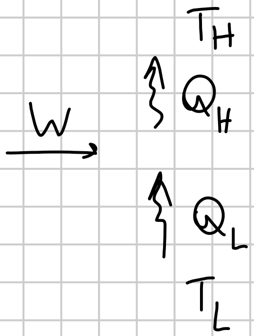
$$\Rightarrow Q_1 = nRT_1 \ln\frac{V_f}{V_i} = nRT_1 \ln 8 = P_1 V_1 \ln 8$$

$$Q_2 = nc_v (T_3 - T_1) = nc_v \cdot \left(\frac{P_3 V_3}{nR} - \frac{P_1 V_1}{nR} \right) = \frac{c_v}{R} \left(\frac{P_1}{8^\gamma} \cdot 8 V_1 - P_1 V_1 \right) =$$

$$= \frac{c_v}{R} P_1 V_1 \cdot \left(\frac{1}{8^{\gamma-1}} - 1 \right)$$

$$\Rightarrow \eta = 1 - \frac{\frac{c_v}{R} P_1 V_1 \left(1 - \frac{1}{8^{\gamma-1}}\right)}{P_1 V_1 \ln 8} = 1 - \frac{\frac{3}{2}\left(1 - \frac{1}{8^{2/3}}\right)}{\ln 8} = \boxed{45.9\%}$$

b) $$W_{prodotto} = \eta Q_1 = \eta P_1 V_1 \ln 8 = 0.459 \cdot 1.013 \times 10^5 \, Pa \cdot 200 \cdot 10^{-3} m^3 \cdot \ln 8 =$$
$$= 19.3 \, kJ$$



$$CoP_{fr} = \frac{|Q_L|}{|W|} = CoP_c = \frac{T_L}{T_H - T_L}$$

$$\Rightarrow |Q_L| = \frac{T_L}{T_H - T_L} \cdot |W|$$

$$\Rightarrow |Q_H| = |Q_L| + |W| = \frac{T_H}{T_H - T_L} |W| \quad (\text{v. anche } CoP_{pompa\ calore})$$

$$|Q_H| = m_{vap} \cdot \lambda_{evap} = \frac{T_H}{T_H - T_L} \cdot |W| \Rightarrow m_{vap} = \frac{T_H}{T_H - T_L} \frac{|W|}{\lambda_{evap}}$$

$$T_H = 373.15 K \qquad T_L = 273.15 K$$

$$\Rightarrow m_{vap} = \frac{373.15 \, K}{100 \, K} \cdot \frac{19.3 \, \frac{kJ}{ciclo}}{2272 \, \frac{kJ}{kg}} = \boxed{31.7 \, \frac{g}{ciclo}}$$